93

Nice Job

MTHSC H311
Test #2
29 October 2008

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You may not use your notes. Please show all of your work. An answer without justification will receive little credit

(1) Let $A = \begin{pmatrix} 1 & 5 & 0 \\ -1 & -4 & 0 \\ -2 & -7 & 9 \end{pmatrix}$ and let $B = \begin{pmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{pmatrix}$.

$\begin{matrix} a & b \\ c & d \end{matrix}$ 2×2 $ad - bc$

a) *(7 points)* Use expansion by cofactors to compute det(A)

$\det A = 0 + 0 + 9\det\begin{vmatrix} 1 & 5 \\ -1 & -4 \end{vmatrix} = 9(-4-(-5)) = 9(1) = 9$

b) *(7 points)* Use row reduction to compute det(B).

$B = \begin{bmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{bmatrix} \xrightarrow[R_3+2R_1]{R_2+R_1} \begin{bmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 3 & -3 \end{bmatrix} \xrightarrow{R_3-3R_2} \begin{bmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$

$\det B = 1 \cdot 1 \cdot 3 = 3$

c) *(6 points)* What is det(AB)?

$\det(AB) = \det A \det B$

$\det(AB) = 27$

20

$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$

TRANSPOSE!

(2) *(9 pts.)* Let $A = \begin{pmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & -1 & 1 \end{pmatrix}$ Compute Adj(A)

$C_{11} = +\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0$ $\quad C_{12} = -\begin{vmatrix} 3 & 0 \\ -1 & 1 \end{vmatrix} = -(3-0) = -3$ $\quad C_{13} = +\begin{vmatrix} 3 & 0 \\ -1 & 1 \end{vmatrix} = 3$

$C_{21} = -\begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix} = -(-2-(-1)) = 1$ $\quad C_{22} = +\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} = (0-1) = -1$ $\quad C_{23} = -\begin{vmatrix} 0 & -2 \\ -1 & 1 \end{vmatrix} = -(0- \; = 2$

$C_{31} = +\begin{vmatrix} -2 & -1 \\ 0 & 0 \end{vmatrix} = 0$ $\quad C_{32} = -\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} = -(0-(-3)) = -3$ $\quad C_{33} = +\begin{vmatrix} 0 & -2 \\ 3 & 0 \end{vmatrix} = 6$

$$\operatorname{Adj} A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & -1 & -3 \\ 3 & 2 & 6 \end{bmatrix}$$

(3) *(12 pts.)* Let $A = \begin{pmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{pmatrix}$. Find a basis for Nul$(A)$, Col$(A)$ and Row(A) non-0 rows

basis Col A: $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} \\ \\ \end{bmatrix} \right.$

$$\begin{bmatrix} 1 & 0 & -5 & 0 & 7 \\ 0 & 1 & -4 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

basis Row A:
$(1, 0, -5, 0, 7)$
$(0, 1, -4, 0, 6)$
$(0, 0, 0, 1, -3)$

$x_1 = 5x_3 - 7x_5$
$x_2 = 4x_3 - 6x_5$
$x_4 = 3x_5$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5x_3 - 7x_5 \\ 4x_3 - 6x_5 \\ x_3 \\ 3x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

basis Nul A: $\left\{ \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$

21

$\det(AB) = \det A \det B$

(4) Recall that $\mathbb{P}_3$ is the vector space of polynomials of degree 3 or less, that is

$$\mathbb{P}_3 = \{p(x) = ax^3 + bx^2 + cx + d : a, b, c, d \in \mathbb{R}\}.$$

Determine which of the following sets is a subspace of $\mathbb{P}_3$ and explain your answer

a) *(6 pts.)* $S = \{q(x) = ax^2 + b : a, b \in \mathbb{R}\}$

0 vec? $0 = 0x^2 + 0$

Yes, subspace, see 3 conditions

Closed w/ add? $u = a_1x^2 + b_1$, $v = a_2x^2 + b_2$

$u + v = a_1x^2 + b_1 + a_2x^2 + b_2 = (a_1 + a_2)x^2 + (b_1 + b_2) \in S$

scalar mult closed? $u = a_1x^2 + b_1$

$cu = c(a_1x^2 + b_1) = (ca_1)x^2 + cb_1 \in S$

b) *(6 pts.)* $T = \{q(x) = ax^3 + 1 : a \in \mathbb{R}\}$

0 vec? $0 = 0x^3 + 1$ doesn't work!

$0 \notin T$

Not a subspace because the 0 vector is not inside

(5) *(14 pts.)* Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}, \quad \text{and} \quad \vec{v} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$$

Compute $[\vec{v}]_{\mathcal{B}}$.

Check

$x = P_B[x]_B$

$[x]_B = P_B^{-1}x$

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$[\vec{v}]_B = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 3 & 8 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

26

dim Col A
rank A + dim Nul A = n
dim Row (?)   cols

(6) *(14 pts)* Consider the following subspace of $\mathbb{R}^4$

$$H = \left\{ \begin{bmatrix} 3a+6b-c \\ 6a-2b-2c \\ -9a+5b+3c \\ -3a+b+c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

Find a basis for H and compute the dimension of H.

LI
Span

$$a\begin{bmatrix} 3 \\ 6 \\ -9 \\ -3 \end{bmatrix} + b\begin{bmatrix} 6 \\ -2 \\ 5 \\ 1 \end{bmatrix} + c\begin{bmatrix} -1 \\ -2 \\ 3 \\ 1 \end{bmatrix}$$

14

$$\begin{bmatrix} 3 & 6 & -1 \\ 0 & -14 & 0 \\ 0 & 23 & 0 \\ 0 & 7 & 0 \end{bmatrix}$$

basis H = $\left\{ \begin{bmatrix} 3 \\ 6 \\ -9 \\ -3 \end{bmatrix} \begin{bmatrix} 6 \\ -2 \\ 5 \\ 1 \end{bmatrix} \right\}$

dim H = 2

(7) *(14 pts.)* Let

$$\begin{aligned} p(t) &= t^3 + 2t^2 + 1, \\ q(t) &= t^2 + 2t + 3, \\ r(t) &= 4t^2 + t, \quad \text{and} \\ s(t) &= 2t^3 + t^2 + t + 5. \end{aligned}$$

$1 + 0t + 2t^2 + t^3$
$3 + 2t + t^2 + 0t^3$
$0 + 1t + 4t^2 + 0t^3$
$5 + t + 1t^2 + 2t^3$

10

Is the set $\{p(t), q(t), r(t), s(t)\}$ linearly independent? If not, exhibit a nontrivial dependence.

$$\begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 2 & 1 & 1 \\ 2 & 1 & 4 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[illegible] weights [illegible] = 0
all solutions [illegible] = 0
are $\begin{bmatrix} -2 \\ -1 \\ 1 \\ 1 \end{bmatrix} x_4$
So, one solution is $\begin{bmatrix} -2 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ ($x_4 = 1$)
which means
$-2p(t) - q(t) + r(t) + s(t) = 0$

No, Linearly dependent

s(t) dependent on r(t)

24

Dependence ?

(8) *(5 pts.)* Suppose that V and W are vector spaces and that $T : V \to W$ is a linear transformation. Show that $\ker(T)$ is a subspace of V

kernel (T)

$T(x) = 0$

subspace if

(1) 0 vector exi[illegible]

(2) closed over vector addition

(3) closed over scalar Multiplic[illegible]

$T: V \to W$

$\ker(T) = \{x \in V : T(x) = 0\}$

[illegible] linear, $T(0) = 0$ [illegible]

$T(0) = T(0+0)$ [illegible]

[illegible]

$0 = 0$

(2) [illegible] $v, w \in \ker(T)$

[illegible]

so $T(v+w) =$ [illegible]

$\Rightarrow v + w \in \ker(T)$

(3) [illegible] $v \in \ker(T)$, $c \in \mathbb{R}$

$T(cv) = c\,T(v) = c \cdot 0 = 0$

$cv \in$ [illegible]

since [illegible] [illegible] subspace [illegible] V

2